The Origin of Universal Power Law of Natural Images

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Abstract. Natural images can fundamentally affect the evolution and development of biological visual systems. Therefore much research has been done on the statistics of natural images. One of the most striking findings is that the orientation averaged power spectrum of natural images follows the universal power law. However little is known the origin of the universal power law. In this paper, the power spectra of natural images were explored under different transforms. We found that the power spectrum was unaffected under histogram equalization transform and a shape transform. However, with a little surprise, it found that the power spectra were affected more by local micro structure than global configure structures. Then a three dimensional and a two dimensional simulations were carried out. The results of the simulations indicated that the universal power law may due to the projection of three dimensional objects with sizes equally distributed. However, the size distribution of two dimensional objects waits further research.

1. Introduction

Natural images can fundamentally affect the evolution and development of biological visual systems. Therefore much research has been done on the statistics of natural images. One of the most striking findings is that the orientation averaged power spectrum S of natural images follows the universal power law [3,4,8,10]

$$S(r) = \frac{A}{r^{2-\eta}}$$

where r is the spatial frequency, and A is a constant. η is the deviation of the universal power law.

This universal power law is generally regarded as a key feature of natural images. Much vision research [1,2,6,7,12] is based on this result. However, one may wonder what the origin of natural images is. Where does it come from? Some researcher [9] tried to explain the origin of the universal power law as the collage region of independent objects which obeyed power law. Clearly, in this case, the researcher only consider the two dimensional situation while we live in a three dimensional world.

In this paper, the power spectra of natural images were explored under different transforms. Then two simulation studies on the origin of the universal power law are presented. In the first simulation, three dimensional objects located uniformly in three dimensional space was studied, it is found that under uniform distribution, the simulated images conform to the universal power law. In the second simulation, different power law of the sizes of the objects are simulated, it found that the simulated images conform to the universal power law with different deviation.

In the next section, the power spectra of natural images are examined with different transform applied. Then the 3D simulation is presented. This is followed by the 2D simulation with different power law of the sizes of the objects. Then a discussion of these simulations is presented. The paper is concluded by a summary.

2. The Natural Images and different Transforms

Around one hundred natural images were collected from the internet. A sample of these images was presented in Figure 1. The orientation averaged mean power spectrum of these images was calculated and found to conform to the universal power law with the deviation -0.32. The power spectrum is presented in Figure 2

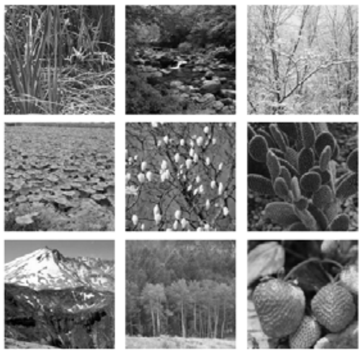


Figure 1: A sample of natural images

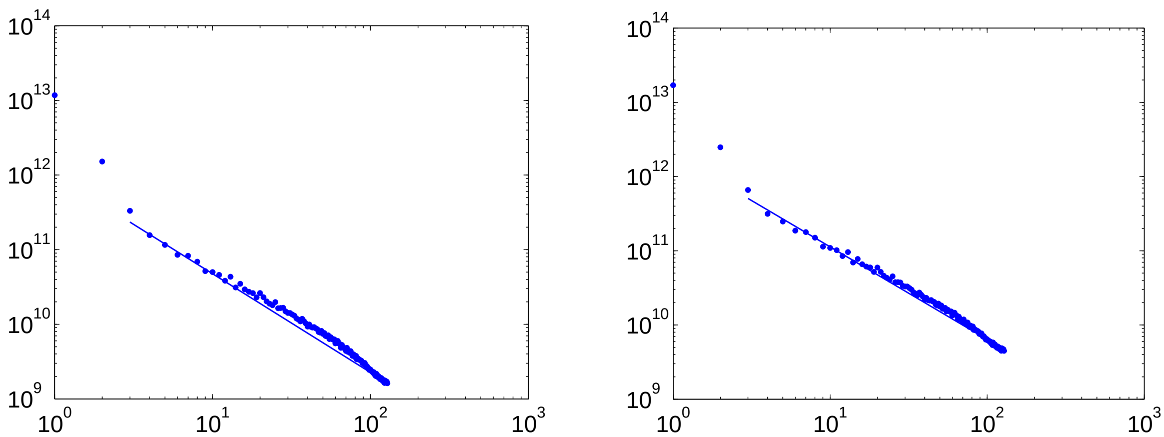


Figure 2: (Left) The orientation averaged power spectrum of the natural images with deviation -0.32 . (Right) The orientation averaged power spectrum of the histogram equalization transformed natural images with deviation -0.25.

One may wonder how the power spectrum changes under different transforms. Here a series of transforms were applied to these natural images. First we applied the histogram equalization to these natural images. The histogram equalization made images to have more contrast while keep the pixel position unchanged. It turned out that orientation averaged power spectrum still conform to the universal power law with the deviation -0.25 (see Figure 2 right).

The next transform applied to these natural images was distortion transform. The distortion transform enlarges or squeezes the images in the horizontal or vertical directions. Two distortion transformed examples are shown in Figure 3.

Figure 3: A sample of original and distorted images

After examine the distortion transformed images, it found that the images still look like natural images. Indeed the orientation averaged power spectrum still conform to the universal power law with the deviation -0.39 (see Figure 4) although the images changed a lot .

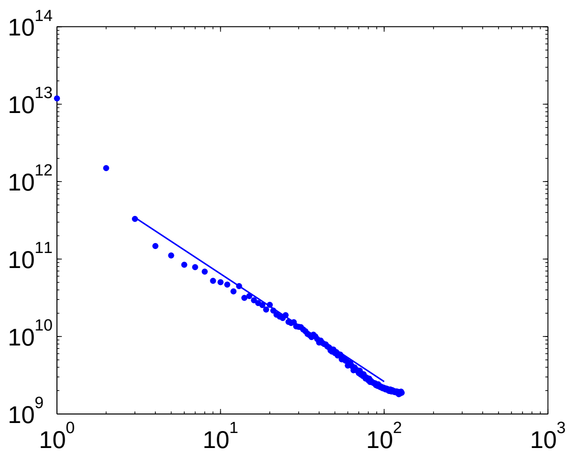
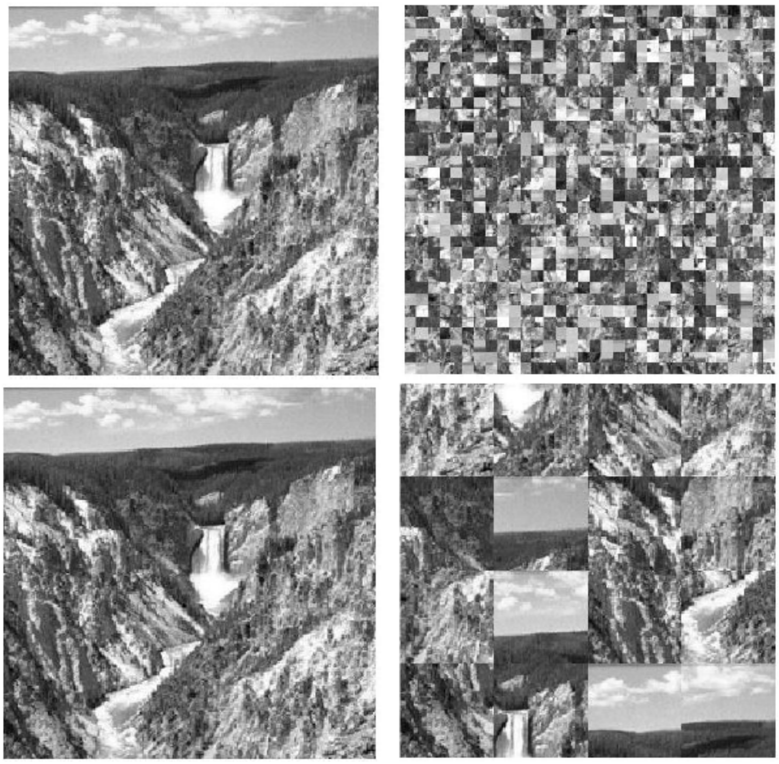


Figure 4: The orientation averaged power spectrum of the distortion transformed natural images with deviation -0.39.

After examining these two transforms, one may wonder how local and global structures affect the power spectra of the natural images. Here the local micro structures were first kept while the global configure structures were randomized. More concretely, the images were first divided to small squares, the microstructure of small squares were kept while their positions were randomized. Figure 5 is two examples of the transformed images.



Original Configure randomized

Figure 5: A sample of randomized images

The corresponding power spectra are presented in Figure 6.

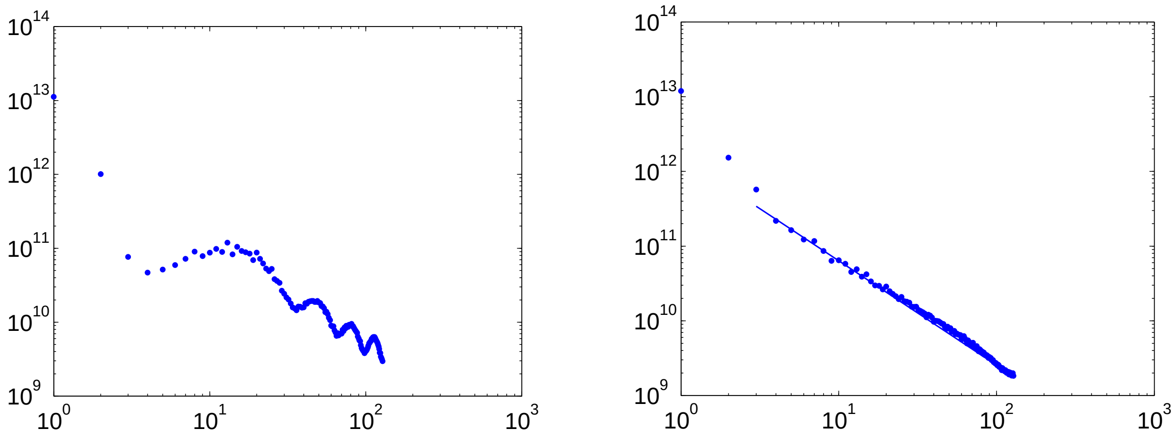
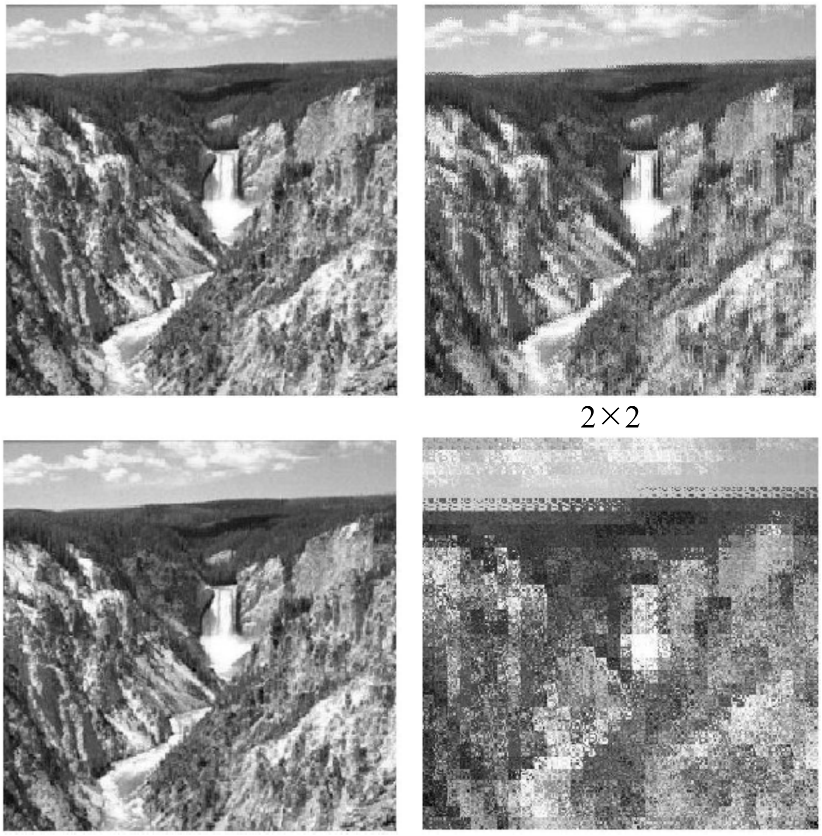


Figure 6: The orientation averaged power spectrum of the configure randomized natural images with deviation -0.37 for 64×64 case while for 8×8 the power spectrum was zig-zag.

It can be observed that for 8×8 configure randomized images, the orientation averaged power spectrum did not conform to the universal power law. Instead there was zig-zag on the power spectrum. In fact, the zig-zag appeared on power spectra of 16×16, 32×32 configure randomized images. However, the zig-zag disappeared for the power spectrum of the 64×64 configure randomized images while 64×64 is consistent with observation that 64×64 was the smallest patches for observable images [11].

The configure randomized transform was a global transform, one may wonder if one kept the global structure while changing the local micro structure, what the power spectra look like? In the following exploration, this kind of local transform were applied. One transformed example is shown in the Figure 7. The corresponding orientation averaged power spectrum was presented in Figure 8.

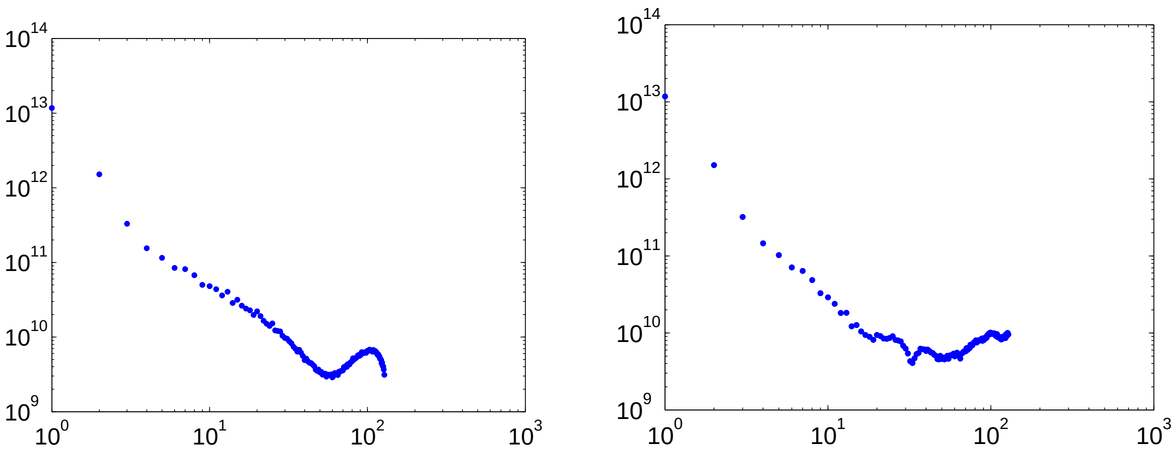


2×2

8×8

Original Local randomized

Figure 7: A sample of original and local randomized images.



2×2 8×8

Figure 8: The orientation averaged power spectrum of the local randomized natural images.

One may be surprised that the local randomized images were just a little blurred while the orientation averaged power spectrum became non-universal power law. In fact, for the 2×2 case, only the very high frequency part did not conform to the universal power law while the other part of the curve still look like a straight line in the loglog coordinates.

3. The Three Dimensional Simulations

Natural images are images of three dimensional space objects such as trees, flowers, rocks, mountains, animals, etc. Research on the origin of the universal power law [9] which primarily worked in the two dimensional space is obviously not feasible in general. So the origin of the universal power law of natural images that it uncovered is not very convincing. Here we worked directly in three dimensional space. It is assumed that objects in the three dimensional space are opaque spheres of different sizes. Using spheres to represent objects can simplify the simulation a lot since all spheres are imaged as circle disks with different sizes.

Assume there was an idealized pinhole camera (see Figure 9 for the camera setup). The camera can be of different configuration with different image sizes and different distance from the pinhole to the images (focal length). Moreover, to be more realistic, the spheres are assumed to be in certain distance from the camera, the distance is not too near or too far away.

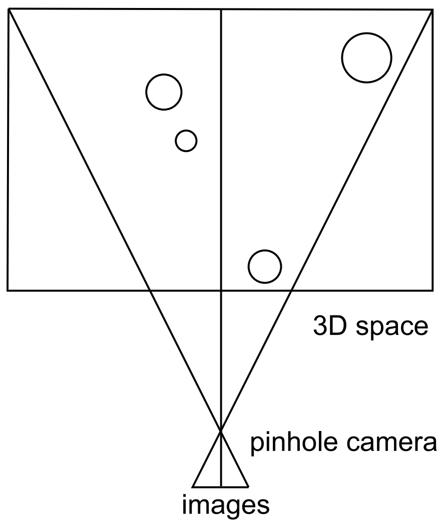


Figure 9: The pinhole camera setup

Then certain amount of the spheres was assumed in the three dimensional space, e.g. assume 20,000 spheres in the three dimensional space. Each sphere could be a random size, located in a random location in this three dimensional space, with different shades (here we only consider the lightness of the images, therefore only shades were considered). To be most generally, the sizes of these spheres were assumed to be uniformly distributed since one has no reason to assume one object size was more likely happen than the other object size. These spheres were then rendered by Computer Graphics algorithm [5].

The objects were first sorted according to the Z dimension (which was the distances from the pinhole camera). Then the objects were rendered from far to near with the near objects occluded the far objects. To avoid the intersection detections which were very difficult, it is assumed that the spheres were represented by different circular disks with infinitesimal thickness when sphere intersection detections were needed. Several of the rendered images were shown in Figure 10.

A series of simulations were conducted. For different simulations, camera setups were modified, e.g. the focal length was increased/decreased, the objects was moved far/near to the camera, etc. In every simulation, around one hundred simulated images were generated and the orientation-averaged power spectrum was computed. It was found that the power spectra conform to the universal power law with deviation -0.14, -0.30, -0.57, -0.67 (Figure 11).

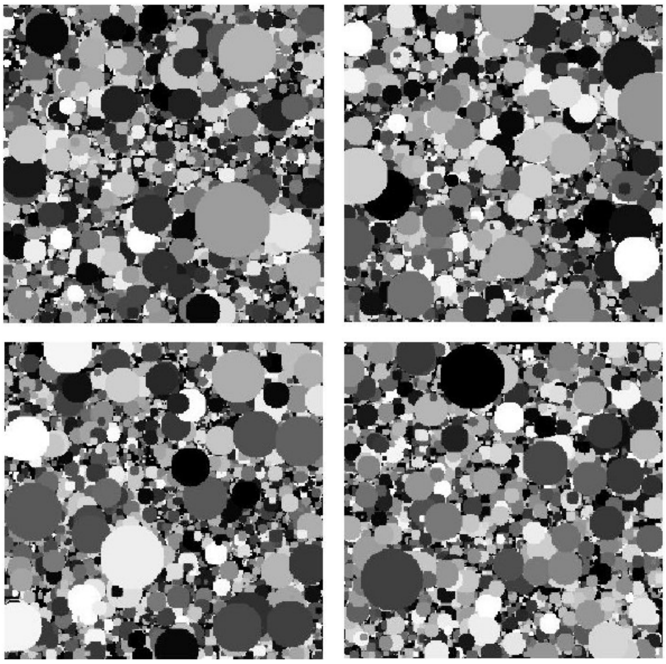


Figure 10: A sample of rendered images of three dimensional space.

The object sizes were assumed to equally distributed in the three dimensional space. However, the sizes of the imaged objects in two dimensional space were more like to be not equally distributed. Big object located far away were imaged as small ones in the two dimensional images while small object located nearby were imaged as big ones in two dimensional images. The reason for this is because the images of three dimensional objects also depend on the third dimensional distance *z*.

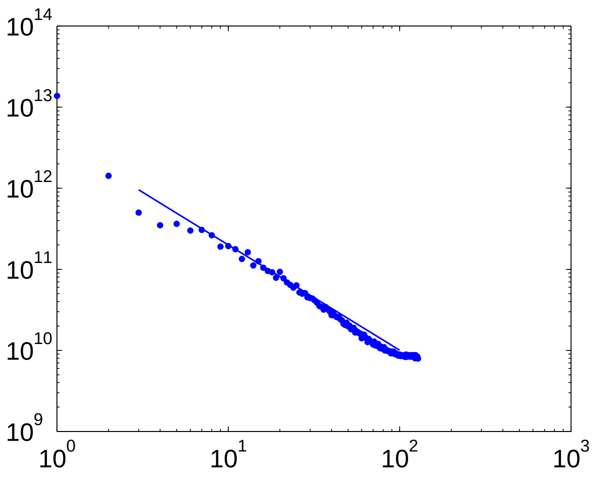


Figure 11: The orientation-averaged power spectrum of the rendered images in log-log plot with different deviation for different camera setups.

The distributions can be calculated in the simulation. One result is presented in Figure 12. In the Figure 12, the horizontal line represented the sizes of the objects in the two dimensional space while the vertical line represented the number of the objects. The numbers of the objects seem follow the $1/size^3$ for small size object while follow 1/size for large size object. Some Research [9] suggested the two dimensional objects follow the power law. One may wonder how the power spectra for different two dimensional object distributions are. This leaded to the next simulation.

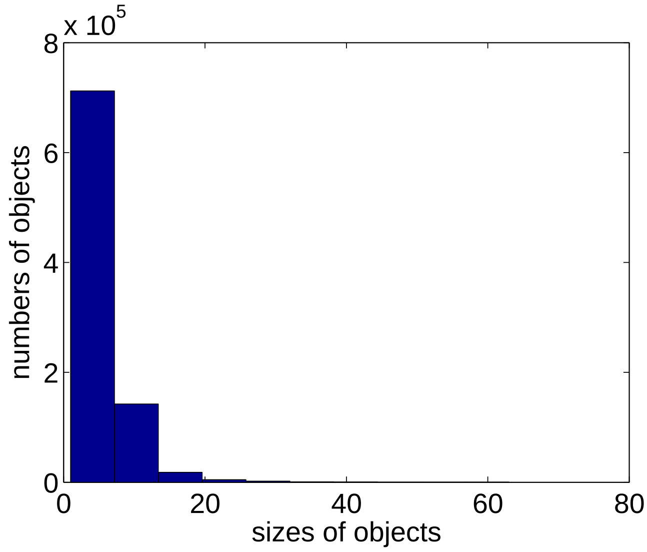


Figure 12: The histogram of the object sizes versus object numbers.

4. The Three Dimensional Simulations

In the two dimensional simulation, the sizes of objects in two dimensional space were assumed to be different power law. It should be noted that in this two dimensional space, there is no the third dimension distance *z* while the sizes of the objects were assumed to follow

$$F(s) = \frac{1}{s^{\gamma}}$$

where s is the size of the objects. Here the γ were assumed to be 3, 2, 1, 0. The simulated images for different γ were presented in Figure 13.

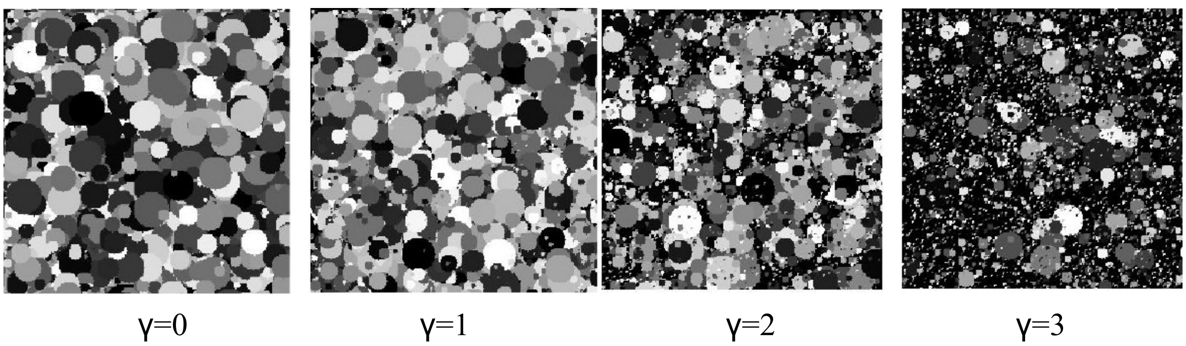


Figure 13: A sample of simulated 2D images.

Around one hundred simulated images are calculated to generate the averaged power spectra for different γ=3, 2, 1, 0$. The deviation for γ=3, 2, 1, 0 are 0.22, -0.17, -0.47, -0.59. One of the power spectra is shown as Figure 14.

One interesting result is that for γ=0, the power law became to be equally distributed for object sizes in the two dimensional space. Still the power spectrum conforms to universal power law. It worth noting that this size distribution is without the third dimension distance *Z*.

It is a little surprised the power spectra of simulated two dimensional images all conformed to universal power law with different deviation. This seems indicate that the universal power law were very robust. It may also indicate that some distribution may not be applied by the natural images although their power spectra conform to the universal power law.

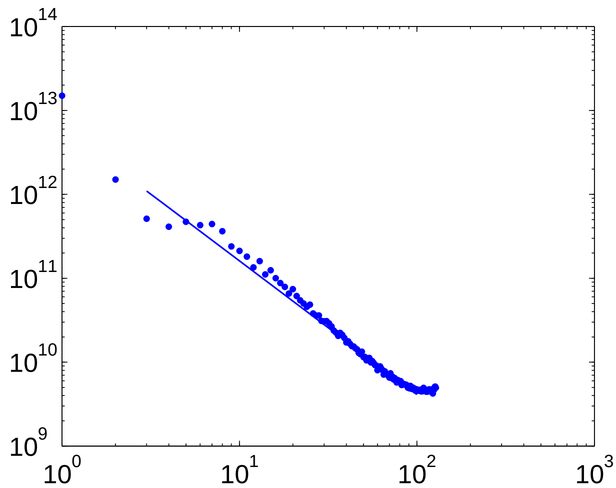


Figure 14: The orientation-averaged power spectrum of the simulated 2D images in log-log plot.

5. Discussion

Natural images can fundamentally affect the evolution and development of the biological vision systems. This leaded to study of the statistics of the natural images. One of the most striking findings is that the power spectrum of natural images conforms to universal power law. However, the questions like what is the origin of the universal power law or what is natural images anyway leave unanswered. Here the power spectra were first analysed under histogram and distortion transforms, it is found that the power spectra were kept. Then we tried to approach these questions from the micro and configure structures. The images were first divided into small patches. Then the micro structures were randomized while the relative positions of these patches were kept. It is found that even for small patches where images changed a little, the power spectrum of the resulted images did not conform to universal power law anymore. We then kept the micro structures and randomize the configure structures of these small patches. It is found that, when patches grew bigger (e.g. 64×64) when more configure structures were kept, the power spectrum of the resulted images again conformed to the universal power law. Interestingly, here only 6.25% configure information of the original images was kept. Furthermore this asymmetry of local micro structures and global configure structure is interesting.

Then we tried to approach this question from two simulation studies. In the first simulation, the objects in 3D space were represented as opaque spheres of different sizes. The sizes and positions of the spheres were assumed to be uniformly distributed, Then images of these spheres were rendered by Computer Graphics algorithm. It is found that the power spectra of the simulated images conformed to the universal power law with deviations around -0.14~-0.67 (corresponding to different camera setups). Moreover the sizes of the circular disks in the simulated images conformed to $1/size^3$ for small disks while conformed to 1/size for large disks. This leaded to the second simulation in 2D which assumed the sizes of circular disks conformed to $1/size^{\gamma}$(γ=0~3). It was found that the power spectra of the simulated images conformed to universal power law with deviation 0.22, -0.17, -0.47, -0.59.

Now one may suggest that the origin of the universal power law is due to the projection of equally distributed independent three dimensional objects to two dimensional images. The universal power law of natural images may just due to this kind of projection. Since statistically one has no reason to assume objects of certain sizes were more likely happen than other sizes of objects, the uniformly distributed sizes of objects were assumed. One interesting question is what is the distribution of the projection of the three dimensional objects? This requires a theoretical analysis. Several power law of the two dimensional objects all gave the power spectra of universal power law. One possibility reason for this is that this may indicate some distribution may not fit the distribution of the three dimensional projection even this power law still gave the universal power law.

6. Conclusion

From this research, one can realize that the power spectra of natural images are robust, e.g., to histogram equalization and distortion transform. However, even a small micro-structure change can lead the power spectra of the changed images did not conform to universal power law. On the other hand, big patches with configure randomization of natural images still conform to the universal power law.

One may also conclude that the origin of the universal power may due to the projection of the three dimensional object to two dimensional images with the size of the three dimensional object equally distributed. Furthermore, that the different power law of two dimensional object size lead to the universal power law may indicate some power law may not be the actual distribution applied by the natural images. What kind of distribution in two dimensional space may be actually applied by the natural images waits for further research.

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